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Branes and wrapping rules

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ABSTRACT

We show that the branes of ten-dimensional IIA/IIB string theory must satisfy, upon toroidal compactification, specific wrapping rules in order to reproduce the number of supersymmetric branes that follows from a supergravity analysis. The realization of these wrapping rules suggests that IIA/IIB string theory contains a whole class of generalized Kaluza–Klein monopoles.

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1. Introduction

It is by now well-understood that branes form a crucial ingredient of string theory. For instance, they have been used to calculate the entropy of certain black holes [1] and they are at the heart of the AdS/CFT correspondence [2]. Often, the presence of a p -brane in string theory can be deduced from the presence of a rank $p+1$ -form potential in the corresponding supergravity theory. It is a relatively new insight that the potentials of a given supergravity theory are not only the ones that describe the physical degrees of freedom of the supermultiplet. It turns out that the supersymmetry algebra allows additional high-rank potentials that do not describe any degree of freedom but, nevertheless, play an important role in describing the coupling of branes to background fields. For maximal supergravity theories, the allowed U-duality representations of these “un-physical” potentials have been classified in [3–5].

A distinguishing feature of the un-physical potentials is that, when considered in different dimensions, they are not related to each other by toroidal compactification. This is unlike the “physical” potentials, including the dual potentials, whose numbers are fixed by the representation theory of the supersymmetry algebra. Indeed, all physical potentials are related by toroidal compactification. Supergravity is therefore not complete in the sense that the lower-dimensional supergravity theories, including the un-physical potentials, do not follow from the reduction of the ten-dimensional supergravity theory. It is this incomplete nature of supergravity

that will lead us to suggest a class of generalized Kaluza–Klein (KK) monopoles in string theory.

In this Letter we will consider the supersymmetric branes of IIA/IIB string theory compactified on a torus, which couple to the fields of the corresponding maximal supergravities. As mentioned above these fields do not only include the physical potentials, i.e. the p -forms with $0 \leq p \leq D-2$ but also the un-physical potentials, i.e. $D-1$ -forms (which are dual to constant parameters) and D -forms (that have no field strength). In [6] we distinguished between *standard* branes, i.e. branes of co-dimension higher than 2, and *non-standard* branes, i.e. branes of co-dimension 2, 1 and 0. While standard branes are automatically classified because their number coincides with the dimension of the U-duality representation of the corresponding field, this is in general not true for the non-standard branes. A prototype example are the 7-branes of IIB string theory: although the supersymmetry algebra closes on an $SL(2, \mathbb{R})$ triplet of 8-forms,¹ only two of them are actually associated to supersymmetric branes [7]: the D7-brane and its S-dual. At present, it has not been worked out what the number of supersymmetric non-standard branes is in a given dimension.

Recently, a step forward in this direction was performed in [8,9]. The strategy of these papers was to analyze the structure of the gauge-invariant Wess–Zumino (WZ) terms and to introduce the following brane criterion: a potential can be associated to a supersymmetric brane if the corresponding gauge-invariant WZ term requires the introduction of worldvolume fields that fit within the

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¹ Actually, the situation in this case is slightly more subtle since the triplet of 9-form curvatures of these potentials satisfies a non-linear constraint. This is a general property of branes of co-dimension 2 which does not play a role in the present discussion.

bosonic sector of a suitable supermultiplet. Decomposing in each dimension $D = 10 - d$ the U-duality representations in terms of T-duality representations as

$$\text{U-duality} \supset \text{SO}(d, d) \times \mathbb{R}^+ \quad (1)$$

one can deduce how the tension T of each brane scales with the string coupling constant g_s in terms of a number α

$$T \sim (g_s)^\alpha. \quad (2)$$

The value of α follows from the \mathbb{R}^+ -weight of the corresponding potential. The analysis of the fields as T-duality representations for each value of α reveals a remarkable recurrence [8,9] at least for the highest values of α . The fundamental fields, that is the fields with $\alpha = 0$, are in all cases a 1-form and a 2-form, which transform respectively as a vector and a singlet under T-duality.² The RR fields, which have $\alpha = -1$, are in all dimensions T-duality spinors of alternating chirality. Finally, the solitonic fields, with $\alpha = -2$, belong to T-duality representations corresponding to antisymmetric tensors of rank zero to four (see [9] for the details).

While the fundamental branes, the D -branes and the standard solitons are in all cases in correspondence with their potentials, the same is not true for the non-standard solitonic branes, and indeed the analysis of [9] reveals that only some components of the representations of the solitonic fields actually lead to supersymmetric branes. The overall result can be nicely summarized by introducing a set of wrapping rules that give the number of fundamental branes (F), D -branes (D) and solitons (S) in dimension D from those in dimension $D + 1$ [6]:³

$$\begin{array}{l} \text{F} \quad \left\{ \begin{array}{l} \text{wrapped} \rightarrow \text{doubled}, \\ \text{unwrapped} \rightarrow \text{undoubled}, \end{array} \right. \\ \text{D} \quad \left\{ \begin{array}{l} \text{wrapped} \rightarrow \text{undoubled}, \\ \text{unwrapped} \rightarrow \text{undoubled}, \end{array} \right. \\ \text{S} \quad \left\{ \begin{array}{l} \text{wrapped} \rightarrow \text{undoubled}, \\ \text{unwrapped} \rightarrow \text{doubled}. \end{array} \right. \end{array} \quad (3)$$

This means that all the branes in a given dimension can be obtained by a simple counting rule starting from the ten-dimensional ones.

The wrapping rule for fundamental branes and D -branes can be easily understood. For fundamental branes, the doubling upon wrapping corresponds to the fact that after compactification on a circle there is an extra fundamental 0-brane resulting from the reduction of a pp-wave, while for D -branes the wrapping rule simply means that the ten-dimensional D -branes (of either IIA or IIB) generate the whole spectrum of D -branes in any dimensions. For *standard* solitons, the doubling is precisely the dual of the one for fundamental branes, and it corresponds to an additional contribution to the number of solitonic $(D - 4)$ -branes due to a wrapped Kaluza–Klein (KK) monopole.

To realize the same dual wrapping rule for the *non-standard* solitons, one needs a class of so-called generalized KK monopoles with 6 worldvolume, n isometry and $4 - n$ transverse directions

Table 1

Forms with $\alpha = -3$ in any dimension. All representations are meant to be irreducible, and the T-duality vector indices $AB \dots$ are always meant to be antisymmetrized. The a, \dot{a} denote chiral and anti-chiral T-duality spinor indices.

$(D - 2)$ -form	$E_{D-2, \dot{a}}$
$(D - 1)$ -form	$E_{D-1, A\dot{a}} + E_{D-1, a}$
D -form	$E_{D, AB\dot{a}} + E_{D, Aa} + 3E_{D, \dot{a}}$

($n = 0, 1, 2, 3, 4$) [9]. Here $n = 0$ corresponds to the NS–NS 5-brane and $n = 1$ to the standard KK monopole. Formally, one can associate to these generalized KK monopoles the following mixed-symmetry fields:

$$\text{IIA/IIB: } D_{6+n, n}, \quad n = 0, 1, 2, 3, 4. \quad (4)$$

The field D_6 is the magnetic dual of the NS–NS 2-form B_2 , while the field $D_{7,1}$, which is the dual of the graviton, is associated to the standard KK monopole. Although this dual graviton field $D_{7,1}$ can only be introduced consistently at the linearized level, it can still be considered as a tool to determine all the lower-dimensional standard solitons by dimensional reduction. Solutions corresponding to more general mixed-symmetry fields have been considered in e.g. [10,11]. The whole set of supersymmetric solitons in any dimensions can be obtained from these mixed-symmetry fields by imposing a restricted reduction rule which states that a supersymmetric brane is only obtained when the n indices on the right of the comma in $D_{6+n, n}$ are internal and along directions that coincide with n of the indices on the left of the comma.

The only ten-dimensional supersymmetric brane which is left aside by this analysis⁴ is the S-dual of the D7-brane of the IIB theory. The tension of this brane scales like $(g_s)^{-3}$ in the string frame. In any dimension below ten, one can deduce the T-duality representations of the $\alpha = -3$ fields by simply looking at the tables in Ref. [9]. This leads to the remarkable result that also for these fields the pattern of T-duality representations is universal, see Table 1.

In this Letter we will analyze the structure of the WZ terms corresponding to the fields in Table 1, in order to determine which of them correspond to supersymmetric branes. As we will see, for the $(D - 1)$ - and the D -forms only the highest dimensional irreducible representation corresponds to a supersymmetric brane. Moreover, we will discover that only a subset of the components of the representations of these fields actually corresponds to a supersymmetric brane. The final result will lead to yet another wrapping rule:

$$\begin{array}{l} \text{wrapped} \rightarrow \text{doubled}, \\ \text{unwrapped} \rightarrow \text{doubled}. \end{array} \quad (5)$$

That is, one obtains the right counting if, going from $D + 1$ to D dimensions, both wrapped and unwrapped branes get doubled.

We will finally show that precisely this counting arises from considering, together with the S-dual of the D7-brane, a specific set of ten-dimensional objects which we generically denote as “generalized KK monopoles”. The same result can be obtained from the IIA point of view, in which case all the branes can be seen to arise from compactifications of generalized KK monopoles as there is no $\alpha = -3$ brane in IIA string theory.

2. A new wrapping rule

We start by reviewing how the S-dual of the D7-brane of IIB string theory satisfies our brane criterion, i.e. the construction of

² In this Letter we are only interested in gauge fields, and we therefore do not consider scalars (which would couple to instantons).

³ Since there are two theories in $D = 10$ (IIA and IIB) it is understood that the wrapping rule is applied as follows when reducing from $D = 10$ to $D = 9$ dimensions: a nine-dimensional “undoubled” brane can be seen as coming from IIA and from IIB, and consistently the set of undoubled branes coming from either IIA or IIB is the same; a nine-dimensional “doubled” brane has only one origin in terms of ten-dimensional branes, which is a IIA or a IIB brane, and the set of doubled branes results from both IIA and IIB, treating each resulting brane as different.

⁴ We are not taking into account the ten-dimensional space-filling branes. These branes can only wrap.

Table 2

By applying the wrapping rule (5) one obtains precisely the number of $\alpha = -3$ supersymmetric branes predicted by the supergravity counting rule (7).

p -brane	IIA/IIB	9	8	7	6	5	4	3
0								64
1							32	448
2						16	192	1344
3					8	80	480	
4				4	32	160		
5			2	12	48			
6		1	4	12				
7	0/1	1	2					

a gauge-invariant WZ term requires the introduction of worldvolume fields that can be associated to an eight-dimensional vector multiplet [9]. Denoting with E_8 the $\alpha = -3$ 8-form potential, and using the notations of [9], one obtains the field strength and gauge transformations

$$K_9 = dE_8 + G_3 D_6 - \frac{1}{2} F_7 C_2,$$

$$\delta E_8 = d\mathcal{E}_7 + G_3 \Lambda_5 - \frac{1}{2} F_7 \lambda_1. \quad (6)$$

Here D_6 is a solitonic field ($\alpha = -2$), C_2 is an RR field ($\alpha = -1$), G_3 is the curvature of C_2 and F_7 is the curvature of D_6 . The explicit expressions can be found in [9]. Furthermore, \mathcal{E}_7 , Λ_5 and λ_1 are the $\alpha = -3$, $\alpha = -2$ and $\alpha = -1$ gauge parameters. One can easily write down a corresponding WZ term, which contains the world volume fields c_1 (associated to the RR field C_2) and d_5 (associated to the solitonic field D_6) together with two transverse scalars. Imposing electromagnetic duality between c_1 and d_5 one obtains a vector plus two scalars, which is the bosonic sector of a vector multiplet on an 8-dimensional world volume.

We now want to repeat the same analysis in any dimension $D = 10 - d$, and determine which of the potentials in Table 1 corresponds to branes by analyzing the worldvolume field content of the corresponding WZ term. According to our brane criterion the worldvolume fields have to form the bosonic sector of a vector multiplet after imposing worldvolume electromagnetic duality and after including the transverse scalars.

The outcome of this analysis, which we present below, will be that the number of supersymmetric branes is

$$(D-3)\text{-branes: } 2^{d-1},$$

$$(D-2)\text{-branes: } d \times 2^{d-1},$$

$$(D-1)\text{-branes: } \binom{d}{2} \times 2^{d-1}. \quad (7)$$

This is summarized in Table 2 for any dimension. It is straightforward to realize that the numbers we get are exactly reproduced by the wrapping rule (5), together with the “initial condition” that there is only one such brane in ten dimensions, which is a IIB 7-brane.

We now proceed by deriving the counting rule (7). We will consider each form occurring in Table 1 separately, starting from the one of lowest rank. We use the notation of [9]. We thus denote with $\mathcal{F}_{1,A}$ the T-duality vector of worldvolume field-strengths associated to the fundamental 1-forms $B_{1,A}$ and to the corresponding worldvolume scalars $b_{0,A}$. The RR fields are denoted with C and their field-strengths with G , while the corresponding worldvolume fields and field-strengths are c and \mathcal{G} . All these objects are in spinor representations of the T-duality group $SO(d, d)$ of alternating chirality. The solitonic fields that we consider are the fields $D_{D-4+i, A_1 \dots A_i}$ for $i = 0, \dots, 4$, and we denote their field

strengths with H . We associate to these fields the worldvolume fields $d_{D-5+i, A_1 \dots A_i}$, with field strength $\mathcal{H}_{D-4+i, A_1 \dots A_i}$. Finally, Γ_A denotes the Gamma matrices of the T-duality group. We refer to the Appendix of [9] for all the properties of these Gamma matrices that will be relevant in the analysis below.

2.1. $(D-2)$ -forms

The $\alpha = -3$ $(D-2)$ -forms always belong to the irreducible spinor representation denoted by the lower index \dot{a} , which is the same chirality as the RR 2-forms $C_{2,\dot{a}}$. We want to determine whether one can write down a WZ term associated to this field that contains the right number of world volume degrees of freedom to form the bosonic sector of a half-supersymmetric vector multiplet. Together with the two transverse scalars resulting from a $(D-2)$ -dimensional world volume in D dimensions, one needs in addition d scalars and one vector. This makes a total of $d+2$, that is $10 - (D-2)$ scalars as appropriate to a $(D-2)$ -dimensional vector multiplet.

We schematically write down the WZ term without computing the actual coefficients. This will turn out in all cases to be enough to determine the supersymmetric branes. The WZ term is

$$E_{D-2} + \sum_{i=0}^1 a_i D_{D-4+i, \{A_i\}} \Gamma^{\{A_i\}} \mathcal{G}_{2-i} + \sum_{i=0}^1 b_i \Gamma^{\{A_i\}} C_{2-i} \mathcal{H}_{D-4+i, \{A_i\}}, \quad (8)$$

where in general $\{A_i\}$ denotes i antisymmetric $SO(d, d)$ vector indices, while all the T-duality spinor indices are understood. Moreover, \mathcal{G} are the field strengths of the $\alpha = -1$ world volume fields $c_{2n,a}$ and $c_{2n+1,\dot{a}}$ and \mathcal{H} are the field strengths of the $\alpha = -2$ world volume fields d_{D-5} and $d_{D-4,A}$. We now count the degrees of freedom, assuming that all the coefficients a_i and b_i are non-vanishing (this will be the assumption that we will make throughout this section). The terms proportional to a_0 and b_0 propagate the fields $c_{1,\dot{a}}$ (the index \dot{a} is fixed) and d_{D-5} , which corresponds to a vector and its dual. The terms proportional to a_1 and b_1 propagate the scalars $c_{0,a}$ and their duals $d_{D-4,A}$. To do the counting, one has to perform a light-cone Gamma matrix analysis similar to the one of [9]. Following [9] we use a light-cone basis $\Gamma_{n\pm}$ for the Gamma matrix. Given that the index \dot{a} is fixed, one can show that for each n only one non-vanishing Gamma matrix appears in the WZ term. This means that in the term proportional to b_1 one has to count only half of the $2d$ indices, which makes d fields d_{D-4} . The same applies for the term proportional to a_1 : the non-vanishing Gamma matrices project the 2^{d-1} components of the field $c_{0,a}$ to d independent components. Imposing electromagnetic duality between the $c_{0,a}$ and the $d_{D-4,A}$ fields, one is left with d scalars. The conclusion is that we expect all the components of the field $E_{D-2,\dot{a}}$ to be associated to supersymmetric branes.

2.2. $(D-1)$ -forms

We now consider the $(D-1)$ -forms of Table 1. It is immediately apparent that the field $E_{D-1,a}$ can never satisfy our criteria since its corresponding WZ term contains far too many worldvolume fields (and in particular it contains the 2-form c_2 which cannot be included in a vector multiplet in general). We are thus led to consider only the field $E_{D-1,A\dot{a}}$ in the irreducible “gravitino” representation of T-duality. The most general WZ term for this field is

$$\begin{aligned}
& E_{D-1,A} + \sum_{i=0}^1 c_i D_{D-3+i,A\{A_i\}} \Gamma^{A_i} \mathcal{G}_{2-i} \\
& + \sum_{i=0}^1 \tilde{c}_i D_{D-3+i,\{A_{i+1}\}} \Gamma^{A_{i+1}} \mathcal{G}_{2-i} \\
& + \sum_{i=0}^1 d_i \Gamma^{A_i} C_{2-i} \mathcal{H}_{D-3+i,A\{A_i\}} \\
& + \sum_{i=0}^1 \tilde{d}_i \Gamma^{A_{i+1}} C_{2-i} \mathcal{H}_{D-3+i,\{A_{i+1}\}} \\
& + E_{D-2} \mathcal{F}_{1,A} - \frac{1}{2d-1} \Gamma_{AB} E_{D-2} \mathcal{F}_1^B, \tag{9}
\end{aligned}$$

where the coefficients c_i and \tilde{c}_i , as well as d_i and \tilde{d}_i , are related so that the resulting expression is Gamma-traceless, and the first term in the last line has been normalized to 1, as one can always do up to field redefinitions.

We now want to count the worldvolume degrees of freedom. We first count the vectors, that correspond to the terms proportional to c_0 , \tilde{c}_0 , d_0 and \tilde{d}_0 . The terms c_0 and d_0 propagate a single vector $c_{1,\hat{a}}$ and a single $(D-4)$ -form $d_{D-4,A}$, which is dual to a vector (the indices \hat{a} and A are fixed). We are going to show below that for a given set of lightlike components inside the gravitino representation these two terms are automatically Gamma-traceless, so that the terms \tilde{c}_0 and \tilde{d}_0 are not needed. More precisely, both terms c_0 and \tilde{c}_0 in the Minkowskian base contribute to give the single term in the lightcone base, and similarly for the other two terms. The absence of these terms guarantees that only one worldvolume vector propagates. It will turn out that these components are exactly those that propagate the right amount of scalars.

To prove the statement above it is convenient to use lightcone coordinates. For each lightlike direction $n\pm$, the corresponding Gamma matrix $\Gamma_{n\pm,\hat{a}}^a$ is vanishing for half of the values of \hat{a} and non-vanishing for the other half. We take the components of $E_{D-1,A\hat{a}}$ to be along the directions for which the corresponding Gamma matrix has only vanishing entries. This forms a $d \times 2^{d-1}$ -dimensional orbit within the gravitino representation. If for instance we take the component $E_{D-1,n+\hat{a}}$ such that the matrix $\Gamma_{n+,\hat{a}}^a$ vanishes, then the matrix $\Gamma_{n-,\hat{a}}^a$ vanishes too, which implies that the term c_0 and the term d_0 are automatically Gamma-traceless along these components. This completes the proof of the statement.

We now count the scalars. We first consider the term c_1 . If the index A of $D_{D-2,AB}$ is say $1+$, the index B can be $1-$ or any of the other $n\pm$ indices, with $n \neq 1$. But if $B = 1-$, then the Gamma matrix in the c_1 term is $\Gamma_{1+,\hat{a}}^a$ which is vanishing for the \hat{a} we are considering. For all the other possibilities, for each n there is always one and only one of the two possibilities $+$ or $-$ for which the corresponding Gamma matrix is non-vanishing. This makes in total $d-1$ possibilities, and for each possibility one picks a scalar field $c_{0,a}$. One thus selects $d-1$ out of the 2^{d-1} scalars. Analogously, for the d_1 term one selects the $(D-3)$ -forms $d_{D-3,1+B}$ such that B is not $1-$ and is only one possibility out of $n\pm$ for each $n \neq 1$. These are $d-1$ $(D-3)$ -forms which are dual to the scalars. Finally, there are two additional scalars. One is the transverse scalar corresponding to a $(D-1)$ -dimensional world volume in D dimensions. The other is $b_{0,A}$ for fixed index A . The previous argument shows again that in lightcone notation and for the lightcone components we are considering the last term in (9) should not be written. We thus have a total of $d+1 = 11-D$ worldvolume scalars, which is the correct amount for a $(D-1)$ -

dimensional worldvolume. To summarize, the number of supersymmetric branes is

$$d \times 2^{d-1}. \tag{10}$$

2.3. D-forms

We now consider the D -forms, corresponding to the last line in Table 1. Again, as in the previous case, it is straightforward to see that only the highest dimensional irreducible tensor-spinor representation can lead to the right worldvolume fields. We thus consider the WZ term

$$\begin{aligned}
& E_{D,AB} + \sum_{i=0}^1 e_i D_{D-2+i,AB\{A_i\}} \Gamma^{A_i} \mathcal{G}_{2-i} \\
& + \sum_{i=0}^1 f_i \Gamma^{A_i} C_{2-i} \mathcal{H}_{D-2+i,AB\{A_i\}} + E_{D-1,[A} \mathcal{F}_{1,B]}, \tag{11}
\end{aligned}$$

where it is understood that each term is projected on its Gamma-traceless part.

We now want to determine the components that give rise to a worldvolume vector multiplet. We consider the indices AB to be of the form $n \pm m \pm$ with $n \neq m$. We take for simplicity the direction $1+2+$. We consider the Gamma matrices $\Gamma_{1+,\hat{a}}^a$ and $\Gamma_{2+,\hat{a}}^a$, and we take the directions \hat{a} such that both Γ_{1+} and Γ_{2+} vanish. These directions are one fourth of the original spinor components, that is 2^{d-3} directions. We wish to show that for each of these directions the corresponding WZ term propagates the right degrees of freedom. This gives a total number of branes equal to

$$\binom{d}{2} \times 2^{d-1}. \tag{12}$$

We first consider the vector. This arises from the terms e_0 and f_0 . Given that the index \hat{a} and the indices AB are fixed, this clearly propagates one vector and its dual. What remains to be seen is that for the components we have selected this is automatically Gamma-traceless. This is automatic, because the Gamma trace corresponds to contracting with $\Gamma_{1-,\hat{a}}^a$ or $\Gamma_{2-,\hat{a}}^a$, which is identically zero for the values of \hat{a} that we have selected. What remains to be considered are the scalars. This corresponds to the e_1 and the f_1 terms. In both terms, the index C in ABC can be $1-$, $2-$ or any $m\pm$ with $m \neq 1, 2$. But in the first two cases, the corresponding Gamma matrix in the WZ term vanishes, so the only possibility is the third, and actually for each m there is only one of the two possibilities $m+$ or $m-$ that gives a non-vanishing result. This selects $d-2$ possibilities. In the e_1 term, the $d-2$ Gamma matrices project on $d-2$ independent combinations of scalars out of the 2^{d-1} scalars $c_{0,a}$, while in the f_1 term this simply selects $d-2$ fields $d_{D-2,1+2+m}$ which are dual to the scalars. To these $d-2$ scalars we have to add the two scalars $b_{0,A}$ and $b_{0,B}$. This gives $d = 10 - D$ scalars, which leads to the right number of degrees of freedom for a D -dimensional worldvolume. It is easy to show that all the terms involving the scalars are automatically Gamma-traceless for the components we have selected.

This concludes our proof of the counting rule (7) which is in line with the new wrapping rule (5).

3. Generalized KK monopoles

As mentioned in the introduction the realization of the soliton wrapping rule (3) requires the introduction of a set of generalized KK monopoles together with the solitonic 5-brane and the standard KK monopole [6]. One can associate the mixed-symmetry

fields given in (4) to these generalized monopoles. Applying a restricted reduction rule to these mixed-symmetry fields yields precisely the same number of solitons that follows from our supergravity analysis.

We now want to perform a similar analysis for the $\alpha = -3$ branes. In particular, we wish to determine which extra objects, which we will generically denote by “generalized KK monopoles”, are needed to realize the new wrapping rule (5). We find that all the branes in Table 2, satisfying the wrapping rule (5), can be obtained from the following set of ten-dimensional mixed-symmetry fields

$$\text{IIA: } E_{8+n,2m+1,n}, \quad n = 0, 1, 2, \quad 2m + 1 \geq n, \quad (13)$$

$$\text{IIB: } E_{8+n,2m,n}, \quad n = 0, 1, 2, \quad 2m \geq n, \quad (14)$$

provided that one uses a similar restricted compactification rule as in the case of the solitons. Explicitly, we have the IIA fields

$$\begin{aligned} E_{9,1,1} \quad E_{8,1} \quad E_{10,3,2} \quad E_{9,3,1} \quad E_{8,3} \quad E_{10,5,2} \quad E_{9,5,1} \quad E_{8,5} \\ E_{10,7,2} \quad E_{9,7,1} \quad E_{8,7} \end{aligned} \quad (15)$$

and the IIB fields

$$\begin{aligned} E_8 \quad E_{10,2,2} \quad E_{9,2,1} \quad E_{8,2} \quad E_{10,4,2} \quad E_{9,4,1} \quad E_{8,4} \quad E_{10,6,2} \\ E_{9,6,1} \quad E_{8,6}. \end{aligned} \quad (16)$$

As an example we show how the counting works in seven dimensions. We have, from IIA,

$$\begin{aligned} E_{9,1,1} &\rightarrow E_{6ijk,i,i} \quad (3) \quad E_{7ij,i,i} \quad (6), \\ E_{8,1} &\rightarrow E_{5ijk,i} \quad (3) \quad E_{6ij,i} \quad (6) \quad E_{7i,i} \quad (3), \\ E_{10,3,2} &\rightarrow E_{7ijk,ijk,ij} \quad (3), \\ E_{9,3,1} &\rightarrow E_{6ijk,ijk,i} \quad (3), \\ E_{8,3} &\rightarrow E_{5ijk,ijk} \quad (1), \end{aligned} \quad (17)$$

where we have used the restricted reduction rule that in $E_{m,n,p}$, with $m \geq n \geq p$, all p indices must be internal and that these internal indices must also occur among the m and n indices. Furthermore, the remaining $n - p$ indices among the n indices are also taken to be internal, and these must also occur among the m indices. For the $E_{m,n}$ fields we use the same restricted reduction rule as for the solitons, see the introduction. Applying these restricted reduction rules gives four 4-branes, twelve 5-branes and twelve 6-branes, which is the correct result, cp. to Table 2. One can easily show that the IIB compactification gives the same result. Similarly, one can show that all the other dimensions work in the same way.

4. Conclusions

In this Letter we showed, by completing our earlier work, that branes whose tension scales as $T \sim (g_s)^\alpha$ for $\alpha = 0, -1, -2, -3$ satisfy the following wrapping rule

$$\text{wrapped} \rightarrow \text{doubled, undoubled, undoubled, doubled}, \quad (18)$$

$$\text{unwrapped} \rightarrow \text{undoubled, undoubled, doubled, doubled}, \quad (19)$$

where the four terms at the right of the arrow correspond to $\alpha = 0, -1, -2$ and -3 , respectively. For $\alpha = 0$ the doubling of branes is due to the reduction of pp-waves. Dirichlet branes, with $\alpha = -1$, have no doubling and are complete by themselves. For standard solitonic branes, with $\alpha = -2$, the doubling is due to the presence of the standard KK monopole. In our previous paper [6]

we suggested that the doubling in the case of non-standard solitons is due to the presence of so-called generalized KK monopoles. Similarly, in the present Letter we introduced a new wrapping rule for $\alpha = -3$ and suggested that the doubling is due to the presence of new objects which we generically called generalized KK monopoles.

At present it is not clear what the precise status of the generalized KK monopoles is. We are able to associate a set of mixed-symmetry fields to them with a restricted reduction rule such that all branes suggested by supergravity are generated upon reduction. The explicit solution for some of the suggested generalized KK monopoles have been given in the literature, see e.g. [10,11]. What is not yet clear is whether a finite energy solution can be obtained, possibly by taking superpositions of such generalized KK monopoles. In the introduction we stated that supergravity is incomplete in the sense that the maximal supergravity theories in different dimensions are not related to each other by toroidal reduction. In some sense the new structure we introduced, generalized KK monopoles or mixed-symmetry fields, takes this incomplete nature of supergravity away. Whether this is merely a book keeping trick or a true physical meaning can be given to the generalized KK monopoles remains to be explored. The role of the very extended Kac–Moody algebra E_{11} [12] in this is intriguing. Not only does E_{11} predict the number of physical and un-physical potentials of maximal supergravity, it also contains as a sub-sector the mixed-symmetry fields (4), (13) and (14) associated to the generalized KK monopoles.

Ten-dimensional string theory does not contain branes with $\alpha < -4$. The IIB theory contains a space-filling brane with $\alpha = -4$, the S-dual of the D9-brane, but space-filling branes can only wrap and therefore no non-trivial wrapping rule can be associated with them. Indeed, for $\alpha = -4$ we do not find a visible pattern like for the higher values of α . Interestingly, lower-dimensional maximal supergravity suggests the existence of non-space-filling branes with $\alpha = -4$. For instance, in $D \leq 6$ dimensions there are domain walls with $\alpha = -4$ and in $D = 3, 4$ dimensions there are branes of co-dimension 2 with $\alpha = -4$. Clearly, such branes do not follow from the reduction of the ten-dimensional IIB space-filling brane and must be the result of reducing a generalized KK monopole with $\alpha = -4$. Similarly, in $D \leq 6$ dimensions maximal supergravity suggests branes with $\alpha \leq -5$ and such branes too must be the result of generalized KK monopoles with $\alpha \leq -5$.

Summarizing, we find that all branes of IIA and IIB string theory, excluding the space-filling branes which should be treated separately, satisfy the wrapping rule (18). The deeper meaning of why branes should satisfy such a simple wrapping rule is unclear to us. It would be interesting to see whether some geometrical interpretation could be given of this rule. In this respect it would be interesting to investigate the doubled wrapping rule we find for the S-dual of the D7-brane and to see whether this could be understood from an F-theory [13] point of view.

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